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"SPECTRORADIOMETRIC CALIBRATION OF THE THEMATIC MAPPER AND MULTISPECTRAL SCANNER SYSTEM"

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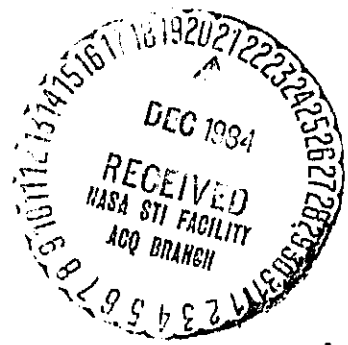

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CHAPTER 2

RADIATIVE TRANSFER THROUGH THE ATMOSPHERE

In this chapter the numeric solution to the transfer of visible and near infrared energy within the atmosphere is discussed. Models of the atmosphere are included. Particular emphasis is given to Rayleigh and Mie theories of scattering, as well as absorption due to atmospheric ozone and water.

The fundamental mathematics for the theory of radiative transfer were developed by Chandrasekhar (1950). He was first to formalize the problem of radiative transfer in a solar illuminated plane-parallel atmosphere, and present a solution in the form of a set of nonlinear equations. He accounted for polarization by adopting the four Stokes parameters to characterize the field. However, even with this framework the solution to these equations remained unsolved for several years. This can be attributed to the fact that the radiance, for a given altitude within the atmosphere and directed in a given direction, is expressed in terms of the radiances incoming from all directions, for that altitude. In practice, the closed form solution, which expresses the field in terms of the known boundary values, cannot be written. The number of equations involved would be overwhelming. Approximations, such as that of single scattering, are often made. This is acceptable provided high accuracy is not a criteria. The LOWTRAN 6 code (Kneizys and associates, 1983), available through the Air Force Geophysics Laboratory, is one such approximate code that is based on the assumption of single scattering.

Using the Gauss-Seidel iterative technique, Herman (1963; Herman and Browning, 1965; Herman, Browning, and Curran, 1971) developed the software required to solve the Chandrasekhar equations. In his method an initial guess is made of the field present after passing through one layer. A solution for successive layers is made using quantities that have been calculated in the previous layer. At the ground a reflectance model, usually lambertian, is used to compute the upwelling radiance. Radiances are then traced moving back up to the top of the atmosphere. Once the radiances at all atmospheric levels have been solved, the process is repeated utilizing updated values of the assumed radiances. All unknowns are changed from their previously calculated values, as the value of the initial unknowns are changed. After several iterations the unknowns converge to a unique solution. This solution is exact in the sense that all orders of multiple scattering are accounted for. Computational accuracy is thought to be limited only by the atmospheric models and input parameters required to run the code.

We use this radiative transfer program for use in the absolute radiometric calibration of Landsat's Thematic Mapper. In the following it is referred to as the Herman Code.

The Equation of Transfer

The attenuation of radiation through some distance ds can be described by the equation

$$dL_\lambda = -k_{T\lambda}\ \rho\ L_\lambda\ ds. \qquad (2.1)$$

Here $k_{T\lambda}$ is the total mass extinction coefficient (in units of area per mass, such as cm^2/gm), ρ is the density of the medium (mass/volume, or gm/cm^3), and L is the radiance (W/cm^2 sr μm) at point s within the medium.

The distance ds is a positive quantity, irrespective of coordinate system. Due to this attenuation, and due to scattering into the beam, radiance varies with distance s. Extinction and density may also vary spatially. In addition, the radiance and mass extinction coefficient vary with wavelength. It is common to identify spectral concentrations with a subscript λ (such as L_λ) and spectral functions as (λ), but for simplicity these notations will not be used further.

The mass extinction coefficient is composed of a scattering term, k_S, and an absorption term, k_a. Thus,

$$k_T = k_S + k_a. \tag{2.2}$$

A related parameter is the volume extinction coefficient, $\beta_T = \rho\, k_T$, which has units of inverse length. Usually one prefers to describe the variation of extinction within the atmosphere in terms of the particle or molecular density. Thus, the radiative transfer equations most often use the mass extinction coefficient, as opposed to the volume extinction coefficient. The former is usually assumed constant throughout the atmosphere. It will only vary spatially with altitude if effects such as pressure broadening, variations of aerosol refractive index, or variations in aerosol radial size distribution occur. Conversely, the parameter β_T varies dramatically with altitude due to its proportionality to density.

To describe the distribution of radiance, normalized to the incoming irradiance, that is scattered from a beam, the phase function $P(\theta)$ is introduced. Here θ is the angle between the incident and scattered beams. Equivalently, $P(\theta,\phi;\theta',\phi')$ describes that radiance scattered from a differential solid angle centered about (θ,ϕ) into a differential solid angle about (θ',ϕ'). In this chapter, the first angles

within parenthesis will be those of the incident beam; the angles which follow the semi-colon are those of the scattered beam. When $P(\theta)$ is used within an integrand, the integration will be with respect to the primed angles. For example, by integrating the phase function over all outgoing angles, $d\omega'$, the total energy lost from a beam through scattering can be computed. This is given by

$$\Delta L_S(\theta,\phi) = -k_\Omega \rho L(\theta,\phi)\, ds \tag{2.3}$$

$$= -k_T \rho\, ds \int_0^{2\pi}\int_0^{\pi} P(\theta,\phi;\theta',\phi')\, L(\theta,\phi) \sin\theta'\, d\theta'\, d\phi'.$$

$L(\theta,\phi)$ is not a function of the scattered angles, and may be placed outside the integral. Using the above equality, the identity

$$\int_0^{2\pi}\int_0^{\pi} P(\theta,\phi;\theta',\phi') \sin\theta'\, d\theta'\, d\phi' = \frac{k_S}{k_T} \equiv \omega_0 \tag{2.4}$$

is made. The ratio k_S/k_T is known as the single scattering albedo, ω_0. It is that fraction of the total attenuation due to scattering for a single collision, and is equal to the integral of the phase function over the scattered angles. A conservative scattering atmosphere is one in which $\omega_0=1$.

The energy balance equation, which summarizes the sources and sinks acting on a beam, can be written

$$dL(\theta,\phi) = k_T \rho\, ds \int_{4\pi} P(\theta',\phi';\theta,\phi)\, L(\theta',\phi')\, d\omega' \tag{2.5}$$

$$+ k_T \rho\, ds\, P(\theta_0,\phi_0;\theta,\phi)\, E + e\rho\, ds - k_T \rho\, L(\theta,\phi)\, ds.$$

The first term is that energy scattered <u>into</u> $d\omega$ due to incoming fields from all directions $d\omega'$, where $d\omega'=\sin\theta'\, d\theta'\, d\phi'$. Note how this integral

over the phase function differs from before. The energy into $L(\theta,\phi)$ from $L(\theta',\phi')$ is computed here, as opposed to the energy out of $L(\theta,\phi)$ into all $L(\theta',\phi')$ as in Equation (2.3). The second term accounts for single scattering out of the solar beam. The incident solar beam has an irradiance E at distance s and propagates along (θ_0,ϕ_0) where $\theta_0>90°$, $\theta_0=\theta_Z+90°$, and θ_Z is the solar zenith angle. The third term accounts for emission within the atmosphere. The spectral parameter e_λ (here denoted only as e) is the emitted spectral radiant flux propagating in an infinitisimal cone containing that direction of propagation, divided by the solid angle of the cone, and normalized with respect to the density of the medium. The final term represents that energy lost due to scattering and absorption processes.

The sources can be readily grouped together by introducing the source function

$$J(\theta,\phi) = \int_{4\pi} P(\theta',\phi';\theta,\phi)\, L(\theta',\phi')\, d\omega' + P_0(\theta_0,\phi_0;\theta,\phi)\, E + e/k_T. \quad (2.6)$$

This is the radiance added to the incident beam from a unit mass of the medium and for a mass extinction coefficient of unity. Throughout the visible and near infrared regions of the spectrum, emission is considered to be negligible. For our application, therefore, the source function will only have contributions from scattering. After dividing both sides by $(-k_T\, \rho\, ds)$, and introducing the source function, Equation (2.5) becomes

$$\frac{dL(\theta,\phi)}{-k_T\, \rho\, ds} = L(\theta,\phi) - J(\theta,\phi). \quad (2.7)$$

The derivative with respect to s is now expanded in terms of derivatives with respect to the x, y, and z axes. Here a cartesian coordinate is defined such that the z-axis is directed upward and the x-axis is directed such that the sun falls within the x-z plane. In addition, the zenith angle θ is defined with respect to an outward normal directed along the z-axis. A beam propagating along $\theta=0^\circ$ is propagating out towards space; a beam directed into the sun will have an azimuth angle of $\phi=0^\circ$. This coordinate system is depicted in Figure 2.1.

Several assumptions are placed on the atmosphere to be modeled. The atmosphere is assumed (a)to be in steady-state (no variations with time), and (b)horizontally homogeneous, which implies a flat earth. (As the following equations use the steady-state assumption, they cannot be used to describe the propagation of a pulsed lidar beam. Here significant changes occur within the atmosphere as the beam travels.) These assumptions imply that there will be no variations in the field or source function along a horizontal plane, and there will be no variations with time. Thus if the generalized function f here represents either L or J, the derivatives df/dx, df/dy, and df/dt will be zero and $df/ds = (df/dz)(dz/ds) = (df/dz)\cos\theta$.

A few definitions may be conveniently introduced here. First let $\mu \equiv |\cos\theta|$. With the sun at a solar zenith angle of θ_z, rays propagating downward from the sun are associated with $-\mu_o=|\cos(\theta_z+90^\circ)|$. While μ itself is always positive, the angle $-\mu$ will be associated with downward propagating beams and μ will be associated with upward directed beams. Secondly, let the optical depth at any height z within the atmosphere be defined as

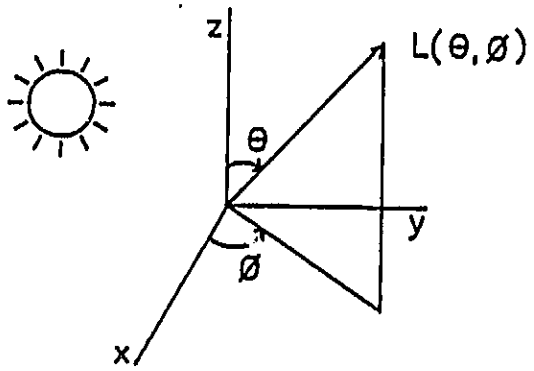

Figure 2.1 Coordinate system used to represent beam directionality.

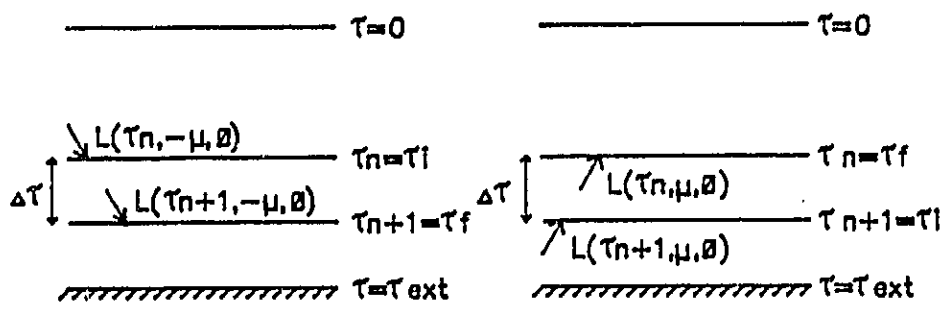

Figure 2.2 Layer nomenclature for beams propagating through the atmosphere.

$$\tau(z) = \int_z^\infty k\ \rho\ dz. \tag{2.8}$$

The optical depth between two altitudes is given as

$$\Delta\tau(z_1,z_2) = \int_{z_1}^{z_2} k\ \rho\ dz \tag{2.9}$$

$$= \tau(z_1) - \tau(z_2).$$

As τ is a positive quantity which monotonically decreases with increas altitude z, the integration within Equation (2.9) will always be set up such that $z_2>z_1$ and hence $\Delta\tau(z_1,z_2)$ will be positive. If z is at ground elevation, and $k=k_T$ the optical depth defined by (2.8) is τ_{ext}, the extinction optical thickness, or total optical depth of the atmosphere.

The attenuation of radiance can now be written in terms of optical depth

$$dL(\theta,\phi) = -k_T\ \rho\ L(\theta,\phi)\ dz/\cos\theta = d\tau\ L(\theta,\phi)/\cos\theta\ . \tag{2.10}$$

Here the substitution $d\tau=-k_T\ \rho\ ds$ and $ds=dz/\cos\theta$ have been made within Equation (2.1). Note $ds=dz/\cos\theta$ is again a positive quantity. For downward directed beams both dz and $\cos\theta$ are negative; for upward directed beams both are positive. This is necessary to assure that dL, as given by Equation (2.10), is always negative. (The beam is attenuated by this term.) Two separate equations are, however, required to express ds as a function of μ, as μ in itself carries no sign. For downward directed beams $ds=-dz/\mu$, while for upward directed beams $ds=dz/\mu$.

The energy balance equation, Equation (2.7), may likewise be rewritten:

downward

$$-\mu \, dL(\tau,-\mu,\phi)/d\tau = L(\tau,-\mu,\phi) - J(\tau,-\mu,\phi) \qquad (2.11a)$$

upward

$$\mu \, dL(\tau,\mu,\phi)/d\tau = L(\tau,\mu,\phi) - J(\tau,\mu,\phi) \ . \qquad (2.11b)$$

For the time being it will be convenient to write separate equations for downward and upward directed beams. Note the radiance and source terms are a function of both altitude z (hence a function of τ) and direction (μ,ϕ). The parameters within parenthesis serve as a reminder of this dependence. The equation is a nonlinear, first order differential equation, subject to the following boundary value conditions. The diffuse radiance incident at the top of the atmosphere is zero, there are no contributions to radiance from below the earth's surface, and the exo-atmospheric solar irradiance is known. That is,

$$\begin{aligned} L(0,-\mu,\phi) &= 0 \\ L(>\tau_{ext},\mu,\phi) &= 0 \\ E_0(-\mu_0,\phi_0) &= \text{known} \end{aligned} \qquad (2.12)$$

The parameter τ_{ext} is the total optical depth at the earth's surface.

Multiplying both sides of Equation (2.11a) by $\exp(\tau/\mu)$ and both sides of Equation (2.11b) by $\exp(-\tau/\mu)$ we obtain

for downward propagation

$$-\mu \, e^{\tau/\mu} \, dL(\tau,-\mu,\phi)/d\tau - L(\tau,-\mu,\phi) \, e^{\tau/\mu} = -\mu \, \frac{d(L \, e^{\tau/\mu})}{d\tau} \qquad (2.13a)$$

$$= -J(\tau,-\mu,\phi) \, e^{\tau/\mu}$$

and for upward propagation

$$\mu \, e^{-\tau/\mu} \, dL(\tau,\mu,\phi)/d\tau - L(\tau,\mu,\phi) \, e^{-\tau/\mu} = \mu \, \frac{d(L \, e^{-\tau/\mu})}{d\tau} \qquad (2.13b)$$

$$= -J(\tau,\mu,\phi) \, e^{-\tau/\mu}$$

Consider a ray as it traverses the layer structure shown in Figure 2.2. The top of the layer is denoted by τ_n and the bottom by τ_{n+1}. These layers are also denoted with τ_i (initial) and τ_f (final), where τ_i can be either τ_n or τ_{n+1}, depending on the direction of propagation. Intermediate altitudes are identified by some τ'. The radiance after propagation is determined by integrating Equation (2.13) between the initial and final τ values. Thus,

downward

$$-\mu \left[L(\tau_{n+1},-\mu,\phi)\, e^{\tau_{n+1}/\mu} - L(\tau_n,-\mu,\phi)\, e^{\tau_n/\mu}\right] \tag{2.14a}$$

$$= -\int_{\tau_n}^{\tau_{n+1}} J(\tau',-\mu,\phi)\, e^{\tau'/\mu}\, d\tau'$$

upward

$$\mu \left[L(\tau_n,\mu,\phi)\, e^{-\tau_n/\mu} - L(\tau_{n+1},\mu,\phi)\, e^{-\tau_{n+1}/\mu}\right] \tag{2.14b}$$

$$= -\int_{\tau_{n+1}}^{\tau_n} J(\tau',\mu,\phi)\, e^{-\tau'/\mu}\, d\tau'$$

Dividing Equation (2.14a) by $(-\mu \exp(\tau_{n+1}/\mu))$ and Equation (2.14b) by $(\mu \exp(-\tau_n/\mu))$, the above can be rewritten:

downward

$$L(\tau_{n+1},-\mu,\phi) = L(\tau_n,-\mu,\phi)\, e^{-(\tau_{n+1}-\tau_n)/\mu} \tag{2.15a}$$

$$+ \int_{\tau_n}^{\tau_{n+1}} J(\tau',\mu,\phi)\, e^{-(\tau_{n+1}-\tau')/\mu}\, d\tau'/\mu$$

upward

$$L(\tau_n,\mu,\phi) = L(\tau_{n+1},\mu,\phi)\, e^{-(\tau_{n+1}-\tau_n)/\mu} + \int_{\tau_n}^{\tau_{n+1}} J(\tau',\mu,\phi)\, e^{-(\tau'-\tau_n)/\mu}\, d\tau'/\mu \ . \qquad (2.15b)$$

These equations can be combined if τ_i and τ_f are introduced.

$$L(\tau_f,\pm\mu,\phi) = L(\tau_i,\pm\mu,\phi)\, e^{-\Delta\tau/\mu} + \int_{\tau_n}^{\tau_{n+1}} J(\tau',\pm\mu,\phi)\, e^{-\Delta\tau'/\mu}\, d\tau'/\mu \qquad (2.16a)$$

where

$$J(\tau',\pm\mu,\phi) = \int_{4\pi} P(\mu',\phi';\pm\mu,\phi)\, L(\mu',\phi')\, d(-\mu')\, d\phi + P(\mu_0,\phi_0;\theta,\phi)\, E \ . \qquad (2.16b)$$

Here $\Delta\tau=\tau_{n+1}-\tau_n$ and $\Delta\tau'=|\tau'-\tau_f|$. The solid angle $d\omega'=\sin\theta'\, d\theta'\, d\phi' = d(\cos\theta')d\phi'$ has been written in terms of μ, or $d\omega'=d(-\mu')\, d\phi'$. Each of the above equations state that the radiance after passing through a layer can be expressed as the initial radiance attenuated by $\exp(-\Delta\tau/\mu)$, plus a contribution from the source function. The source function adds a contribution at each altitude τ', but is attenuated due to the $\Delta\tau'$ between τ' and the final layer.

Numeric Solution

To evaluate the above radiative transfer equation, the integrals within Equations (2.16a) and (2.16b) are replaced with an equivalent sum of integrals with smaller differences between the limits of integration. The new limits are defined such that the parameters J, P, and L can be approximated as constants within the $\Delta\tau$, $\Delta\theta$, and $\Delta\phi$ intervals. They are put outside the integrals, an evaluation is made, and a solution is obtained.

INTRODUCTION

This is the eighth quarterly report on Contract NAS5-27382 entitled "Spectroradiometric Calibration of the Thematic Mapper and the Multispectral Scanner System".

On 26 - 28 October 1984, we made a successful trip to White Sands, New Mexico, and took a variety of radiometric measurements on the morning of the Landsat 5 Thematic Mapper overpass. The sky that morning was cloud free, our sites were dry although many other areas at White Sands were covered by several centimeters of water due to the unusually rainy Fall.

Barnes multiband radiometer data were collected for a 4 X 4 pixel area and two fractional pixel areas of slightly higher and lower reflectances than the larger area. Helicopter color photography was obtained of all the ground areas. This photography will allow us to make a detailed reflectance map of the 4 X 4 pixel area and we will be able to register it to the TM imagery to an accuracy of better than half a pixel. Spectropolarimeter data were also collected of the 4 X 4 pixel area from the helicopter. In addition, ground based solar radiometer data were collected to provide spectral extinction optical thickness values. The uncorrected Thematic Mapper image, in CCT form, of White Sands for that date is expected in a few days. The completion of the calibration of the TM for that date awaits receipt of the CCT. A description of the data reduction and calibration will be given in the next quarterly report.

The remainder of this report consists of a description of the radiative transfer theory used in the development of the Herman code which we use in predicting the TM entrance pupil spectral radiances from the ground based measurements just mentioned. The theory is then fundamental to the measurement program we are conducting at White Sands. It has been written up by C.J. Kastner and is also to be included as a chapter in her Ph.D. dissertation.

In evaluating the integral over optical depth, Equation (2.16a), the radiative transfer between layers τ_n and τ_{n+2} is considered. This allows the average value of the source term to be taken as that at the midpoint of the interval, namely that at τ_{n+1}. After factorin out this constant value and evaluating $J(\tau_{n+1},\pm\mu,\phi) \int_{\tau_n}^{\tau_{n+2}} e^{-\Delta\tau'/\mu}\, d\tau'/\mu$, Equation (2.16a) becomes

$$L(\tau_f,\pm\mu,\phi) = L(\tau_i,\pm\mu,\phi)\, e^{-2\Delta\tau/\mu} + J(\tau_{n+1},\pm\mu,\phi)\,(1-e^{-2\Delta\tau/\mu}) \qquad (2.17)$$

where the interval $\Delta\tau$ is still defined as that between τ_n and τ_{n+1}, and τ_f and τ_i are separated by a $2\Delta\tau$ thickness.

In turn, $J(\tau_{n+1},\pm\mu,\phi)$ is evaluated by replacing the integrals within (2.16b) with sums over the finite differences $\Delta\mu$ and $\Delta\phi$. That is,

$$J(\tau_{n+1},\pm\mu,\phi) = \sum_{k=1}^{2\pi/\Delta\phi} \left[\sum_{j=1}^{\pi/\Delta\theta} P(\mu'_j,\phi'_k;\pm\mu,\phi)\, L(\tau_{n+1},\mu'_j,\phi'_k)\, (-\Delta\mu')_j \right] (\Delta\phi')_k \,. \qquad (2.18)$$

As an example, let $\Delta\theta=10^\circ$ and $\Delta\phi=30^\circ$ (actually their radian equivalents). Then,

$$\begin{aligned} &j = 1, \ldots, 18 \qquad (2.19)\\ &\quad \theta'_j = j\Delta\theta - \Delta\theta/2 = [5^\circ, 15^\circ, \ldots, 175^\circ]\\ &\quad \mu'_j = \cos\theta'_j\\ &\quad (-\Delta\mu')_j = \cos((j-1)\Delta\theta) - \cos(j\Delta\theta)\\ &\qquad\qquad = [\cos 0^\circ - \cos 10^\circ, \ldots, \cos 170^\circ - \cos 180^\circ]\\ &k = 1, \ldots, 12\\ &\quad \phi'_k = k\Delta\phi - \Delta\phi/2 = [15^\circ, 45^\circ, \ldots, 345^\circ]\\ &\quad (\Delta\phi')_k = \Delta\phi = 30^\circ * \pi \text{ rad}/180^\circ \end{aligned}$$

Note P and L have been taken out of the integral over the finite limits $\Delta\theta$ and $\Delta\phi$, and replaced with their values at the midpoints of these finite differences.

At the beginning of each iteration through the atmsophere, the radiances at level τ_i are required. This is achieved by considering the transfer of radiation through only a single $\Delta\tau$ layer,

$$L(\tau_i,-\mu,\phi) = (1-e^{-\Delta\tau/\mu}) \left[\sum_{k=1}^{2\pi/\Delta\phi} \sum_{j=1}^{\pi/\Delta\theta} P(\mu'_j,\phi'_k;-\mu,\phi) \right. \tag{2.20}$$

$$\left. L(0,\mu'_j,\phi'_k)\ (-\Delta\mu')_j\ \Delta\phi + P(-\mu_0,\phi_0;-\mu,\phi)\ E_0 \right] .$$

On the first pass all upwelling radiances $L(0,\mu'_j,\phi'_k)$, or that energy being reflected out of the atmosphere and into space, are assumed zero. On successive passes, those values computed in previous iterations are assumed. At all times the downwelling radiances $L(0,\mu'_j,\phi'_k)$, at the top of the atmosphere, are assumed zero. This is a statement that the only energy entering the atmosphere is from the solar irradiance E_0. Similarly, at the bottom of the atmosphere the radiances at τ_{ext-1} are computed from those at a single $\Delta\tau$ layer below. At τ_{ext} the radiances are those reflected from the surface. They are computed by multiplying the sum of the diffuse and direct downwelling irradiances by ρ/π.

In choosing a numeric value for the layer thickness, $\Delta\tau$, Herman (1963) used a statistical analysis to compute the probability that scattering within a layer would be due to single scattering alone. The $\Delta\tau$ interval must be small enough to neglect variations of the source term, which is equivalent to requiring that L and E remain approximately constant over the interval. This is likely if a photon has a small probability of undergoing two or more scattering events. Conversely, $\Delta\tau$ must not be so small as to make the computation time excessive. A value of $\Delta\tau$=0.02 was chosen. Here, approximately 96% of the scattered radiation is associated with a single collision. Since the effective depth

of the atmosphere is $\Delta\tau\mu$, a greater percentage of multiple scattering occurs at larger zenith angles. As θ approaches 90^0, this error builds up rapidly. Calculations down to 85^0 can, however, be made without introducing any serious errors.

Polarization

The Herman Code which we have used to date does not account for changes in polarization as a ray propagates through the atmosphere. The code can, however, be easily modified to do so. Preliminary studies have indicated that the nonpolarization code is accurate enough, given the atmospheric conditions we have encountered at White Sands to date, for our calibration work. For this reason the studies within this dissertation have been made using the original Herman code, which is both easier and faster to run. For completeness, the theory behind the polarization code is discussed here.

To be as accurate as possible, the radiative transfer equation must describe the state of polarization of a scattered field, as this field generally has undergone a change in polarization compared to that of the incident field. To describe this state, the amplitude of the electric field components along two orthogonal directions, as well as the phase difference between these components, are required. For example, let E_l and E_r be the parallel and perpendicular components, defined with respect to a reference plane. This reference plane is chosen as that containing the incident and scattered beams. Then,

$$\begin{aligned} E_l &= a_l \exp(-i\delta_l) \exp(i(\omega t - k \\ E_r &= a_l \exp(-i\delta_r) \exp(i(\omega t - kz), \\ \delta &= \delta_l - \delta_r \end{aligned} \tag{2.21}$$

As an alternative to requiring that the amplitudes A_l and A_r, and the

phase difference δ be known, the state of polarization may be represented by the four Stokes parameters introduced by Sir George Stokes in 1852. These have the advantage of all having the same dimension, that of an irradiance. The four parameters are

$$\begin{aligned} I_l &= E_l E_l^* = a_l^2 \\ I_r &= E_r E_r^* = a_r^2 \\ U &= 2\,\mathrm{Re}(E_l E_r^*) = 2\,a_l a_r \cos\delta \\ V &= 2\,\mathrm{Im}(E_l E_r^*) = 2\,a_l a_r \sin\delta \end{aligned} \tag{2.22}$$

where the asterisk denotes the complex conjugate has been taken.

Referring to Figure 2.3, the state of polarization can be represented by an ellipse, which in turn is described by the Stokes parameters. Let χ be the angle between the direction of the major axis and the l direction. Knowing χ is equivalent to knowing the plane of polarization, or that plane through the direction of propagation and ray containing the maximum electric field vector. Also, let the ellipticity be represented by the angle β whose tangent is the ratio of the lengths of the major and minor axes. It can be shown, as in Chandrasekhar (1950), that

$$\begin{aligned} I &= I_l + I_r \\ Q &= I_l - I_r \\ \tan 2\chi &= U/Q \\ \sin 2\beta &= V/I \end{aligned} \tag{2.23}$$

Therefore, the parameters I_l, I_r, U, and V represent the irradiances in two perpendicular directions within a plane transverse to the direction of propagation, the plane of polarization, and the ellipticity of the electromagnetic wave. With these, all quantities relevant to the description of the state of polarization are determined. In addition, the percentage polarization is given as

$$P = \sqrt{Q^2+U^2+V^2}/I \tag{2.24}$$

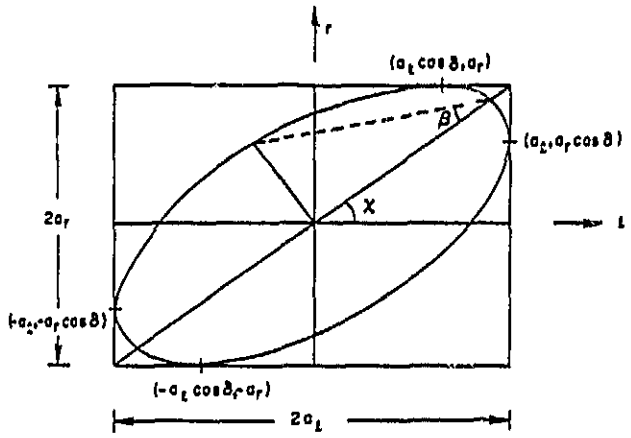

Figure 2.3 Representation of elliptical polarization (from Liou, 1980).

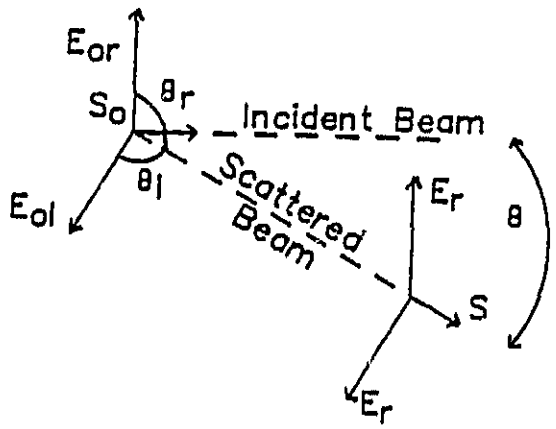

Figure 2.4 Dipole scattering.

For unpolarized light U=V=0, as the time average over sinδ and cosδ are zero, and $I_l = I_r$. The polarization P equals zero, as expected. One example of unpolarized light is that which is initially from the sun, although the light becomes partially polarized after scattering within the atmosphere. Conversely, for a completely polarized beam $I^2=Q^2+U^2+V^2$, and P=1.

The expressions

$$E_l = S_l E_{0l} \qquad (2.25)$$
$$E_r = S_r E_{0r}$$

are next utilized to determine the four Stokes parameters of the scattered field. S_l and S_r relate the magnitude of the scattered fields E_l and E_r to that of the incident fields E_{0l} and E_{0r}. They are functions of the angle between the incident and scattered directions of propagation, and, as will be shown later, differ amongst the Rayleigh and Mie particles. By substituting Equation (2.25) into (2.22), the four Stokes parameters are determined

$$\begin{aligned} I_l &= S_l S_l^* I_{0l} \\ I_r &= S_r S_r^* I_{0r} \\ U &= U_0 \, Re(S_l S_r^*) + V_0 \, Im(S_l S_r^*) \\ V &= U_0 \, Im(S_l S_r^*) + V_0 \, Re(S_l S_r^*) \end{aligned} \qquad (2.26)$$

In much the same way as Equations (2.16a,b) represents the transfer of radiant flux within the atmosphere, they likewise can represent the transformation of the Stokes parameters as the beam they represent undergoes scattering within the atmosphere. A few modifications are, however, required. First, from Equation (2.26) it can be shown that not all the scattered Stokes parameters are independent of

each other. To account for this, the radiative transfer equation is rewritten in matrix form. That is,

$$L_p(\tau_f,\pm\mu,\phi) = L_p(\tau_i,\pm\mu,\phi)\, e^{-\Delta\tau/\mu} + \int_{\tau_n}^{\tau_n+\Delta\tau} J_{pq}(\tau',\pm\mu,\phi)\, e^{-\Delta\tau'/\mu}\, d\tau'/\mu \qquad (2.27a)$$

and

$$J_{pq}(\tau',\pm\mu,\phi) = \int_{4\pi} P_{pq}(\mu',\phi';\pm\mu,\phi)\, L_q(\tau',\mu',\phi')\, d\omega' \qquad (2.27b)$$

$$+ P_{pq}(-\mu_o,\phi_o;\pm\mu,\phi)\, E_{oq}\, e^{-\tau'/\mu_o} .$$

Here p and q are related to one of the four Stokes parameters, and P_{pq} is a 4x4 matrix. Thus, the p^{th} component of radiance is determined by summing the source function over the four incoming Stokes components, i.e. q=1,2,3,4.

The matrix P_{pq} cannot be written directly from Equation (2.26). As the equation of radiative transfer traces components relative to a vertical plane within the atmosphere, not the scattering plane, a coordinate transformation must be performed. The most general such scattering phase matrix has been given by Sekera (1955). It takes the form

$$P_{pq} = \begin{matrix} A_{11}A_{11}^* & A_{12}A_{12}^* & \mathrm{Re}(A_{11}A_{12}^*) & -\mathrm{Im}(A_{11}A_{12}^*) \\ A_{21}A_{21}^* & A_{22}A_{22}^* & \mathrm{Re}(A_{21}A_{22}^*) & -\mathrm{Im}(A_{21}A_{22}^*) \\ 2\,\mathrm{Re}(A_{11}A_{21}^*) & 2\,\mathrm{Re}(A_{12}A_{22}^*) & \mathrm{Re}(A_{12}^*A_{21}+A_{11}A_{22}^*) & -\mathrm{Im}(A_{21}A_{12}^*+A_{1.}A_{22}^* \\ & & &) \\ 2\,\mathrm{Im}(A_{11}A_{21}^*) & 2\,\mathrm{Im}(A_{12}A_{22}^*) & \mathrm{Im}(A_{11}A_{22}^*-A_{12}^*A_{21}) & \mathrm{Re}(A_{11}A_{22}^*-A_{21}A_{12}^*) \end{matrix} \qquad (2.28)$$

The quantities A_{pq} are given by

$$\begin{aligned} A_{11} &= T_1 \cos\Delta\phi + T_2 \cos\psi \\ A_{12} &= (\mu' T_1 + \mu T_2) \sin\Delta\phi \\ A_{21} &= (\mu T_1 + \mu' T_2) \sin\Delta\phi \\ A_{22} &= T_1 \cos\psi + T_2 \cos\Delta\phi . \end{aligned} \qquad (2.29)$$

Here μ and μ' are directional cosines of the incident and scattered beams (measured, as before, from the local vertical). The angle $\Delta\phi$ is defined as the difference between the azimuth angles of the incident and scattered beams. Furthermore,

$$\cos\psi = (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2} + \mu\mu' \cos\Delta\phi \qquad (2.30)$$
$$T_1 = (S_1 - XS_r)/(1-X^2)$$
$$T_2 = (S_r - XS_1)/(1-X^2)$$
$$X = \cos\theta = \mu\mu' + (1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos\Delta\phi$$

and S_1, S_r are the proportionality constants defined in Equation (2.25). It is to be noted that all the functions within Equations (2.28) through (2.30) are defined with respect to the angles θ, θ', and $\Delta\phi$.

Rayleigh Scattering by Molecules

Both molecules, whose size are on the order of 10^{-4} μm, and aerosols, ranging from 0.01 to 10 μm, are responsible for scattering within the atmosphere. Molecular scattering in the visible and near ir, where $2\pi r << \lambda$, can be characterized by a simple scattering law due to Lord Rayleigh (J.W. Strutt, third Baron of Rayleigh). In 1872 he derived the scattering law, which now bears his name, using the elastic-solid ether theory. He predicted that scattering varies inversely as the fourth power of the wavelength, and so explained the blue color of the sky. In 1899 Rayleigh revised his derivation to use the electromagnetic theories of Maxwell and Hertz. Thus, the dependance of scattering on refractive index was determined. The scattering law has since undergone one slight revision, to account for molecular anisotropy. This was done in the 1920's, shortly after some scattering experiments made by Rayleigh's son demonstrated the need for this modification. A complete development of the Rayleigh scattering law is given in texts such as McCartney (1976).

Highlights of its development are given here.

Dipole Scattering

To begin with, the mechanical oscillator model of the atom is used. A binding force is characterized by a spring which induces a linear restoring force to the electron as it is displaced. Such a displacement occurs when a molecule is subject to an applied electric field, E_0. An induced dipole moment p=ex is created, where e is the charge on an electron, and x is the displacement. This electric dipole oscillates synchronously with the field, and in turn produces the scattered wave. The new field is proportional to (1)the acceleration of the electron, (2)$\sin\theta$, where θ is the angle between the dipole moment and direction of observation, and it is inversely proportional to R, the distance from the dipole. It has an amplitude

$$E = \frac{\omega^2 \, p_0 \, \sin\theta \, \sin\omega(t-R/c)}{4\pi\epsilon_0 \, c^2 \, R} \, . \qquad (2.31)$$

Because of the $\sin\theta$ dependance, the dipole cannot radiate along the axis of the dipole. The maximum dipole moment p_0 is found by solving the equation of motion for the maximum electron displacement:

$$p_0 = e \, x_0 = \frac{e^2 \, E_0}{m(\omega_0^2-\omega^2)} = \frac{(n^2-1) \, 3 \, \epsilon_0 \, E_0}{(n^2+1) \, N} \, . \qquad (2.32)$$

Here, ω_0 is the resonant frequency of oscillation, equal to $(k/m)^{1/2}$ where k is the restoring force on the electron.

The latter equality within Equation (2.32) utilizes the Lorenz-Lorentz expression to substitute for the molecular parameters. Now n, the refractive index of the gas in bulk form, and N, the number of dipole oscillators per unit volume, are used. The refractive index of air

molecules considered here are found to be nearly 1, real, and vary as a function of wavelength. This wavelength dependance is given by Edlen (1953) as

$$(n-1) \times 10^8 = 6432.8 + \frac{2{,}949{,}810}{146-\lambda^{-2}} + \frac{25{,}540}{41-\lambda^{-2}} \quad . \tag{2.33}$$

For example, $n=1.000293$ at $\lambda=0.55$ μm. Both the Lorenz-Lorentz and Edlen expressions are derived in many discussions on the dispersion of electromagnetic waves, as in Liou (1980).

The Rayleigh expressions assume that scatterers have resonant frequencies far above the visible and infrared spectral regions. Thus they are pure scatterers, and absorb no energy. Such an assumption is valid for nitrogen and oxygen molecules, which are responsible for 99% of molecular scattering. There are, however, molecules that do have an imaginary component to their refractive index at those wavelengths of interest (i.e., they have resonant frequencies near those frequencies corresponding to visible light). The effects of scattering from these species can be overlooked without loss of accuracy, as they compose such a small fraction of the atmospheric gases. Ozone and water vapor are two such absorbers. (The columnar amount of ozone is typically on the order of 0.35 cm-atm. This implies that there will be only 0.35 cm of ozone within a 1 cm^2 atmospheric column of air, in which there are several kilometers of atmospheric scatterers.)

The irradiance produced at a distant point R from the dipole is given by the Poyning vector $\overline{S}$,

$$\overline{S} = c\ \varepsilon_0\ \langle E^2 \rangle \quad . \tag{2.34}$$

The mean of E^2 is found by substituting a factor of 1/2 for $\sin^2\omega(t-R/c)$, and using Equations (2.31) and (2.32) for the electric field strength. To

remove the dependence of scattering on distance R, the intensity, I (Watts/sr), is computed instead. The intensity at distance R is found by multiplying the irradiance $\overline{S}$ by R^2 (since $I=d\phi/d\omega=d\phi/dA \times dA/d\omega$, and $dA=R^2 d\omega$). Hence,

$$I(\theta) = \frac{\pi^2 \varepsilon_0 c \sin^2\theta (n^2-1)^2 E_0^2}{2 N^2 \lambda^4} \quad . \tag{2.35}$$

In addition to the previous equations, the substitution $\omega=2\pi c/\lambda$ and $(n+2)^2=9$ (since $n=1$) have been made in writing Equation (2.35).

Cross Section

The scattering cross section of a gas molecule is defined as that cross section of an incident wave, acted on by the molecule, having an area such that the irradiance flowing across it is equal to the total irradiance scattered in all directions. Thus,

$$\sigma = \frac{\int_{4\pi} I(\theta')\, d\omega'}{c\, \varepsilon_0\, E_0^2/2} \quad . \tag{2.36}$$

Using Equations (2.35), $d\omega'=2\pi \sin\theta' d\theta'$, and $\int_0^{\pi} \sin^3\theta' d\theta'=4/3$, the cross section is obtained. To this, the correction factor $(6+3\delta)/(6-7\delta)$ must be added. This is done to account for molecular anisotropy, which prevents the dipole moment from aligning itself exactly with the electric vector of the primary wave. Thus,

$$\sigma_{Ray} = \frac{8\pi^3(n^2-1)^2}{3 N^2 \lambda^4} \frac{6+3\delta}{6-7\delta} \quad . \tag{2.37}$$

Gucker and Basu (1953) have determined that $\delta=0.035$.

Rayleigh Optical Depth

The volume scattering coefficient for molecules, β_{Ray}, gives the fractional amount of flux scattered in all directions, for a unit volume of gas. As the scattered field from a collection of dipoles add incoherently, the angular coefficient for a unit volume is just N (molecules/volume) times the cross section given by (2.37), or $\beta_{Ray}=\sigma_{Ray}$ N. (Also, the mass extinction coefficient is found to be $k_{Ray}=\sigma_{Ray}$ $N/\rho=\sigma_{Ray}/m$, or cross section per unit mass, where m is the mass of the molecule). Using the definition of optical depth, Equation (2.9), the Rayleigh component of optical depth is determined

$$\tau_{Ray} = \sigma_{Ray} \int_z^{\infty} N(z)\, dz. \qquad (2.38)$$

Model values of the molecular number density as a function of altitude can be found in the U.S. Standard Atmosphere-1962 (see, for example, Valley (1965), or Elterman (1968)), and are given here in Table 3.2.

The tabulated values of mass density, ρ, or number density, N, refer to air at sea-level temperature and pressure. It is desirable to compute the scattering coefficients at nonstandard values of temperature, pressure, and altitude. This is done using the equation of state for an ideal gas ($P=\rho RT$, P being atmospheric pressure, R the universal gas constant, and T the temperature on the Kelvin scale). Thus,

$$\rho = \rho_0 \frac{P}{P_0} \frac{T_0}{T}, \qquad (2.39)$$

where ρ_0, P_0, and T_0 are defined at standard atmosphere conditions.

In using the Herman code to model the atmosphere, τ_{Ray} is determined using measured values of atmospheric pressure. At ground level,.

$$\tau_{Ray} = \frac{8\pi^3(n^2-1)^2}{3\,\lambda^4\,N^2}\;\frac{6+3\delta}{6-7\delta}\;N_c\;10^{16}\;\frac{P}{P_0}\,, \qquad (2.40)$$

where

n = refractive index as given by Equation (2.33)
λ = wavelength in μm
N_s = molecular number density at sea level for a standard atmosphere
$= 2.547 \times 10^{19}\ cm^{-3}$
N_c = columnar number density
$= 2.154 \times 10^{25}\ cm^{-2}$
$\delta = 0.035$
P_0 = 1013.25 mbar, or 29.92 in Hg
P = measured atmospheric pressure, same units as P_0.

Using this formulism at λ=0.55 μm, for example, τ_{Ray}=.098.

Phase Function

The angular dependance on scattering is expressed in terms of the phase function P(θ). This function is defined as the ratio of the radiance into a given direction, to the average radiance in all directions. Thus, the integral of the phase function must be normalized to unity, as there is no absorption by Rayleigh molecules, and

$$\int_{4\pi} P(\theta)\, d\omega' = \omega_0 = 1\ . \qquad (2.41)$$

To derive the phase function for the scattering of unpolarized light by Rayleigh particles, the incident electric field vector is decomposed into two orthogonal components. As before, let E_l and E_r represent those scattered components parallel and perpendicular to a reference plane, and let E_{0l} and E_{0r} be the corresponding incident components. The reference plane is taken as that containing the incident and scattered waves, and the scattered wave is deviated from the incident wave by an angle θ. For each of these two components (i=l or r), the

scattered radiance is found using

$$L(\theta_1) = I(\theta)\,(6+3\delta)/(6-7\delta)\,N\,ds = 3/8\pi\,\sigma_{Ray}\,N\,ds\,\sin^2\theta_1\,S_{\theta i} \quad . \quad (2.42)$$

In arriving at this expression, it is noted that as $I(\theta)$ gives the intensity scattered from a single molecule, $I(\theta)N\,ds$ gives the radiance scattered from a volume of gas. After accounting for anisotropy, the intensity is expressed in terms of σ_{Ray} by using (2.37). The incident irradiance $c\varepsilon_0 E_0^2/2$ is then expressed as $S_{\theta i}$.

The angles θ_1 and θ_r can readily be expressed in terms of the scattering angle, θ. With reference to Figure 2.4, it is shown that $\theta_1=\pi/2-\theta$, and $\theta_r=\pi/2$. Hence, the total scattered radiance is given as

$$L = L_r + L_1 = 3/8\pi\,\sigma\,N\,ds\,L_{\theta r} + 3/8\pi\,\sigma\,N\,ds\,\cos^2\theta\,S_{\theta r} \quad . \quad (2.43)$$

But, as the incoming field is unpolarized, $L_{\theta r}=L_{\theta 1}=L_\theta/2$. Equation (2.43) becomes

$$L = \sigma\,N\,ds\,3/16\pi\,(1+\cos^2\theta)\,s_0 \quad . \quad (2.44)$$

Removing the angular dependence and multiplying by a scaling factor to satisfy (2.41), the phase function for Rayleigh scattering of unpolarized light is found to be

$$P(\theta) = 3/16\pi\,(1+\cos^2\theta) \quad . \quad (2.45)$$

This expression is the Rayleigh component of the phase function used within Equation (2.16). It is thus an important parameter in the calculations of the transfer of radiant flux within the atmosphere.

Because the perpendicular and parallel components are not scattered equally, the resulting radiance will be partially polarized. Although the scattered perpendicular component is independent of the angle θ, the parallel component follows a $\cos^2\theta$ dependance. Thus, if the observation direction is at 90° to the incoming beam, the scattered light

will be completely polarized. The scattered energy is symmetric about the incident beam, and equal amount of energy are sent into the forward and backward hemispheres. If there were only single scattering within the atmosphere, and the atmosphere was composed purely of Rayleigh particles, the skylight everywhere from a 90° angle from the earth-sun line would be completely polarized. This perfectly polarized light is never observed in practice, as the scattering from aerosols, the reflected light from the surface, and the anisotrophy of air molecules themselves cannot be neglected.

If these polarization changes are to be traced through the atmosphere, the matrix form of the phase function is required. For Rayleigh scattering, this becomes (2.46)

$$P_{pq} = 3/8\pi \begin{bmatrix} \cos^2\psi & \mu^2\sin^2\Delta\phi & \mu\cos\psi\ \sin\Delta\phi & 0 \\ \mu'^2\sin^2\Delta\phi & \cos^2\Delta\phi & -\mu'\sin\Delta\phi\ \cos\Delta\phi & 0 \\ -2\mu'\cos\psi\ \sin\Delta\phi & 2\mu\sin\Delta\phi\ \cos\Delta\phi & -\mu\mu'\sin^2\Delta\phi+\cos\psi\ \cos\Delta\phi & 0 \\ 0 & 0 & 0 & \cos\psi\ \cos\Delta\phi+\mu\mu'\sin^2\Delta\phi \end{bmatrix}$$

Mie Scattering

To describe scattering by particles of arbitrary size the equations developed by Mie (1908) are universally used. In developing this theory it was necessary to make the simplifying assumption that the scattering particles were isotropic spheres. Even so, the derivation is complex, using Maxwell's equations, a boundary value analysis, and expansion of the emerging wave in terms of a series of Bessel and Legendre polynomials. The equations can be approximated by the first term of the Mie series for small particles. For this case, however, Rayleigh theory yields an equivalent result with significantly fewer computations. Thus, the term Mie scattering is loosely used to refer to

the scattering by larger particles which do not lie within the Rayleigh regime.

A complete development of Mie theory is given by Stratton (1941) and van de Hulst (1957). The scattered light is again found by breaking the incident beam into components perpendicula. and parallel to the scattering plane. The scattered intensities $I_r(\theta)$ and $I_l(\theta)$ are proportional to the functions

$$i_r = |2\pi/\lambda\ S_r|^2 = \left| \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \pi_n + b_n \tau_n) \right|^2 \qquad (2.47a)$$

$$i_l = |2\pi/\lambda\ S_l|^2 = \left| \sum_{n=1}^{\infty} \frac{2n+1}{n(n+1)} (a_n \tau_n + b_n \pi_n) \right|^2 . \qquad (2.47b)$$

Each function is found as the sum of an infinite series. Defining the size parameter as $\alpha=2\pi r/\lambda$ where r is the the radius of the particle, it is found that the number of terms required for convergence is somewhat greater than α, for $\alpha>1$. The amplitudes of the n^{th} electric partial wave and the n^{th} magnetic wave are given by the complex coefficients a_n and b_n. These are

$$a_n = - \frac{j_n(m\alpha)[\alpha\ j_n(\alpha)]' - j_n(\alpha)[m\alpha\ j_n(m\alpha)]'}{j_n(m\alpha)[\alpha\ h_n^{(2)}(\alpha)]' - h_n^{(2)}(\alpha)[m\alpha\ j_n(m\alpha)]'} \qquad (2.48a)$$

$$b_n = - \frac{j_n(\alpha)[m\alpha\ j_n(m\alpha)]' - m^2\ j_n(m\alpha)[\alpha\ j_n(\alpha)]'}{h_n^{(2)}(\alpha)[m\alpha\ j_n(m\alpha)]' - m^2\ j_n(m\alpha)[\alpha\ h_n^{(2)}(\alpha)]'} . \qquad (2.48b)$$

With air as the incident medium, the parameter $m=n_{re}(1-n_{im}i)$ is related to both the real and imaginary components of the refractive index within the sphere. Spherical Bessel and Hankel functions are denoted by j_n and h_n respectively, and primes denote derivatives with respect to the indicated

arguments. Thus the coefficients a_n and b_n are determined from the particle characteristics, but are independent of the scattering angle θ. This latter dependance is expressed through the functions π_n and τ_n and involve the first and second derivatives of Legendre polynomials:

$$\pi_n(\cos\theta) = \frac{d(P_n(\cos\theta))}{d(\cos\theta)} \tag{2.49a}$$

$$\tau_n(\cos\theta) = \cos\theta\ \pi_n(\cos\theta) - \sin^2\theta\ \frac{d(\pi_n(\cos\theta))}{d(\cos\theta)} \tag{2.49b}$$

When the particle is illuminated by plane-polarized light, the intensity of the scattered light is given by

$$I(\theta) = E_0\ \frac{\lambda^2}{4\pi^2}\ (i_r\ \sin^2\psi + i_l\ \cos^2\psi)\ . \tag{2.50}$$

Here E_0 is the irradiance of the incoming beam, ψ is the angle of the electric vector from the scattering plane, and i_r and i_l are as defined in (2.47). For a particle illuminated with a wave whose electric vector is perpendicular to the plane of observation, $\psi=90^0$ and the scattered beam is polarized in the perpendicular direction. Conversely, an incident beam described by $\psi=0$ is polarized parallel to the scattered plane, as is the scattered beam. For illumination by an unpolarized beam, the scattered intensity is given by

$$I(\theta) = E_{0r}\ \frac{\lambda^2}{4\pi^2}\ i_r + E_{0l}\ \frac{\lambda^2}{4\pi^2}\ i_l \tag{2.51}$$

$$= E_0\ \frac{\lambda^2}{8\pi^2}\ (i_r + i_l)$$

where $E_{0r}=E_{0l}=E_0/2$.

The angular distribution of the scattered field is depicted in Figure 2.5. Here the solid lines refer to scattering from a perpendicular component of the electric vector, and the dashed lines represent scattering from a parallel component. For $\alpha<0.1$ the distribution is

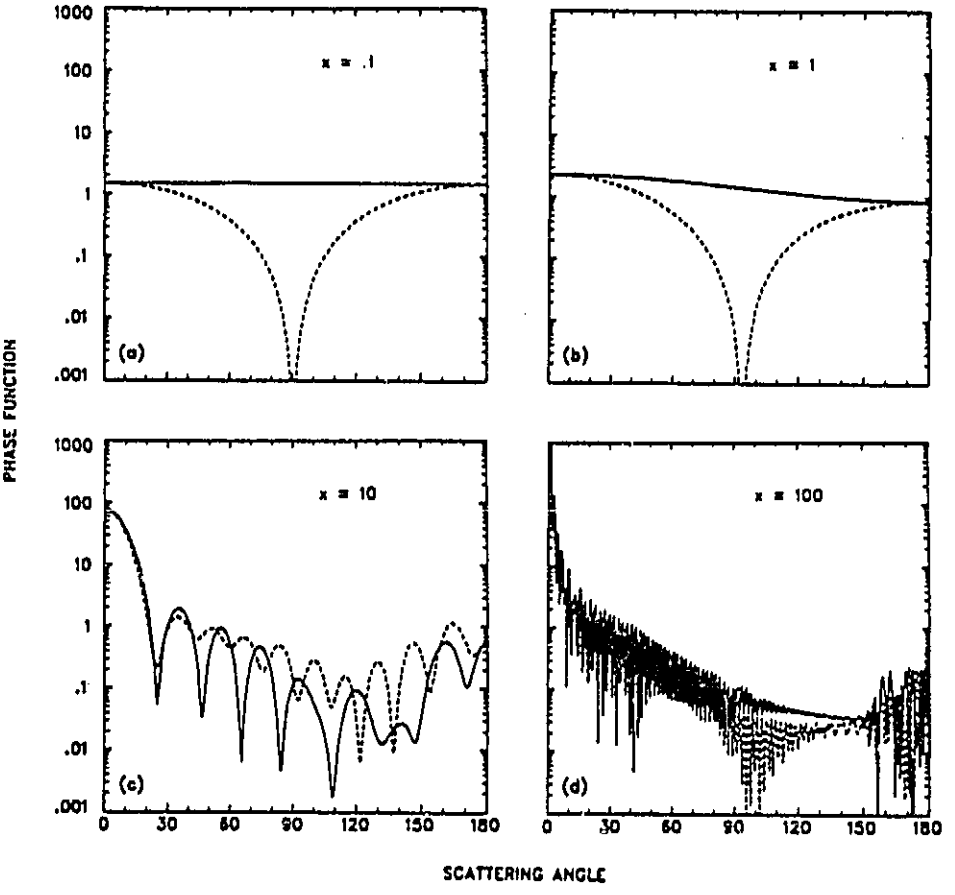

Figure 2.5 Mie scattering from particles of fixed size (from Grams, 1978).

identical to that predicted from Rayleigh theory. There is a $\cos^2\theta$ dependance in the scattered parallel component but no angular variation in the perpendicular component. As α increases (or particle size for a given wavelength), a larger portion of the energy is scattered into the forward direction. If the particle size approaches the wavelength of light, side lobes begin to appear. The frequency of this structure increases with α and the width decreases.

The cross section of a Mie scatterer can now be defined. Unlike scattering from a Rayleigh particle, some energy is lost due to absorption as a beam impinges upon a Mie scatterer. The cross section σ_{Mie} must include the effects of this loss. Defining σ_{sc} as the component which accounts for the energy scattered into all directions, and σ_{abs} the component which accounts for absorption, we have

$$\sigma_{Mie} = \sigma_{sc} + \sigma_{abs} \quad . \tag{2.52}$$

Using Equation (2.50) and assuming unpolarized illumination, σ_{sc} is computed from

$$\sigma_{sc} = \int I(\theta)\, d\omega/E_0 = \lambda^2/8\pi^2 \int_{4\pi} (i_r + i_l)\sin\theta\, d\theta\, d\phi \tag{2.53}$$

$$= \lambda^2/2\pi \sum_{n=1}^{\infty} (2n+1)(|a_n|^2 + |b_n|^2) \quad .$$

The total cross section can likewise be expressed in terms of the Mie coefficients,

$$\sigma_{Mie} = \lambda^2/2\pi \sum_{n=1}^{\infty} (2n+1)\, \mathrm{Re}(a_n + b_n) \quad . \tag{2.54}$$

In the above the expansions in terms of a_n and b_n do not easily follow.

Reference is given to van de Hulst (1957, section 9.32) for more details.

It is noted that the above cross sections are defined for a particle of fixed radius r. Absorption and scattering within the atmosphere are, however, processes which depend on the cumulative effects of many particles within a large size range. This distribution is expressed in terms of a size distribution n(r). It is the normalized number of particles per unit interval of radius per unit volume, hence

$$\int_0^\infty n(r)\, dr = 1 \quad . \tag{2.55}$$

To determine the properties of light scattered from a polydispersion (collection of particles of different radii), the functions i_r and i_l within (2.53) are integrated over the size distribution. The scattered energy from such a distribution of particles is very different from that depicted above. The most obvious difference is that the scattered distribution is a much smoother function of wavelength. A few examples of this are given in Figure 2.6. To compute these curves a log-normal particle distribution was assumed. A mean radius of r_m=1 μm, standard deviation 2 μm, wavelength λ=0.633 μm, and real refractive index n_{re}=1.525 were assumed. Curves (a) and (b) give the the results for a parallel and perpendicular incident electric vector, respectively. In curves (c) and (d) the molecular scattering contributions have been added. Each example has been computed at several values of the imaginary component of refractive index. As n_{im} increases the light scattered into angles greater than θ=15° decreases. The most significant result of increasing the imaginary refractive index, however, is the increase in absorption. This change can be expressed through the parameters σ_{abs} or ω_o, the

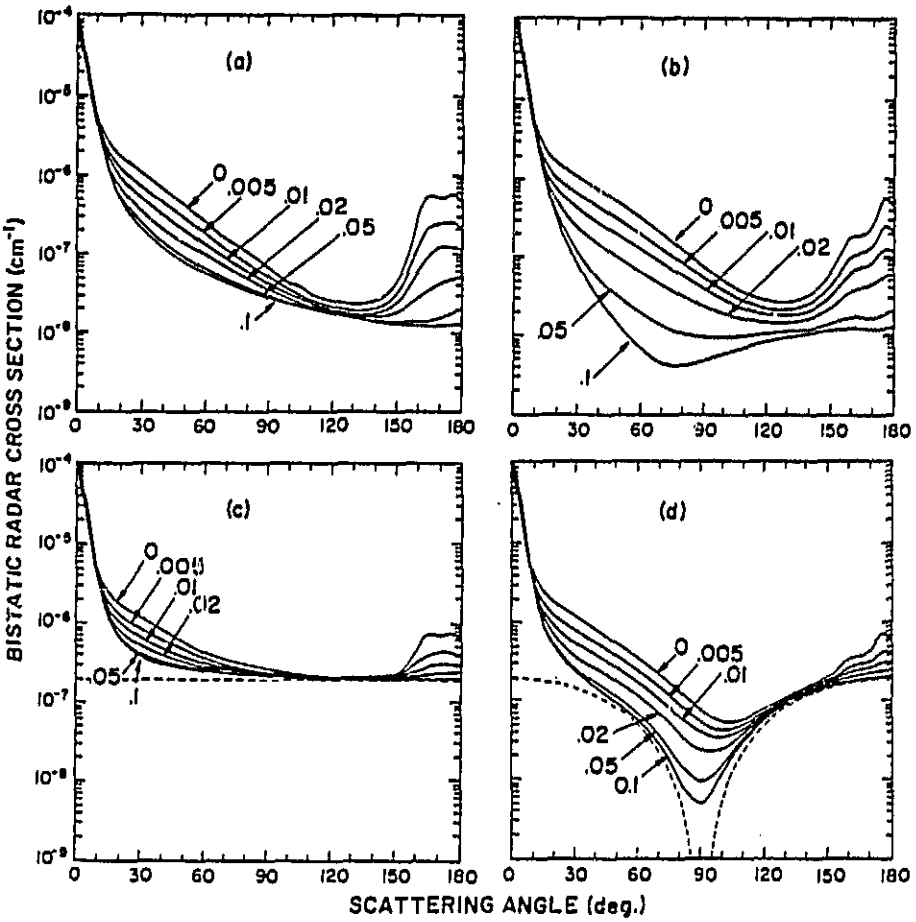

Figure 2.6 Mie scattering by atmospheric polydispersions (from Grams, 1978).

single scatter albedo.

By integrating the cross section over the size distribution function, the optical depth of the atmosphere can be determined. Defining N(z) as the total number of particles per unit volume at altitude z,

$$\tau_{Mie} = \int \int N(z)\, \sigma_{mie}(r)\, n(r)\, dr\, dz \quad . \tag{2.56}$$

Note that the size distribution is taken as constant with respect to altitude. This is usually assumed the case, for lack of better data. More will be said about the radial size distribution function n(r) and the vertical distribution N(z) in the sections to follow.

By integrating the cross section over the size distribution, the phase function for Mie scattering can also be found. To most readily see this, let us define the angular scattering cross section $\sigma_{sc}(\theta)$ as the cross section of the incident wave acted on by the particle, having an area such that the irradiance flowing across it is equal to the intensity scattered into angle θ. The cross section σ_{sc} defined earlier is equal to the angular cross section integrated over all outgoing angles. With this, the phase function is defined as

$$P(\theta) = \frac{\int \sigma_{sc}(\theta,r)\, n(r)\, dr}{\int \sigma_{Mie}(r)\, n(r)\, dr} \quad . \tag{2.57}$$

From this definition it is apparent that the integral of the phase function over all solid angles will not necessarily be equal to one. It will be equal to ω_0, the single scatter albedo, and equal to one only if

there is no absorption of energy by the particle (in such a case $\sigma_{Mie}=\sigma_{sc}$). The greater the imaginary component of refractive index, the smaller ω_0 will be, hence a smaller fraction of energy will be scattered, a greater fraction absorbed.

For illumination by unpolarized light

$$P(\theta) = \frac{\lambda^2/8\pi^2 \int n(r)(i_r + i_l)\, dr}{\int \sigma_{Mie}(r)\, n(r)\, dr} \qquad (2.58)$$

To run that version of the Herman Code which accounts for polarization, the phase function must be written in matrix form. This is done by using S_r and S_l, as defined in (2.47), within Equations (2.28)-(2.30).

Another parameter, closely related to the cross section, that is commonly referred to in the literature is the efficiency factor Q, defined as the cross section of a particle divided by the geometric cross-sectional area of that particle, πr^2. If the scattering efficiency factor is ploted versus the size paramter α, Q_{sc} obtains a maximum value of 2 and converges in an oscillatory fashion to a value of one for high α. This implies that the particle can, at times, interact with an incident wavefront greater than its own geometric area. This is explained through diffraction effects, in which diffracted flux is directed into a small angle centered about the forward direction of the incident flux.

Dave Code

To compute the Mie parameters discussed above, a Fortran computer program written by Dave (1969) is used. This program is incorporated

into one of the subroutines within the Herman Code. Two similiar Dave codes exist, one using an upward recurrence relationship (starting with a value of $a_0(m\alpha)$, successively higher values are computed), and one using a downward recurrence relationship. In the code which uses an upward recurrence algorithm, any error in the first term will propagate and for large enough α the results oscillate wildly around the correct value. For this reason the downward recurrence routine is preferred. It does, however, more storage, and required 10-20% more run time. Both codes require double precision arithmetic, and output results accurate to 6 significant figures (with the one exception mentioned above, where oscillations occur).

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